Solving Two-scale Transport Laws During Frying of Foods Using Comsol Multiphysics

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Abstract: Numerous novel equations such as generalized Darcy’s law based fluid transport equations for various phases; near-equilibrium equation governing phase change from liquid water to vapor or vice versa; and generalized Laplace law of heat transfer were obtained using hybrid mixture theory. Despite the appearance of novel terms in equations, they could be implemented in Comsol Multiphysics using the general PDE option. The integration subdomain variables were used to obtain the average values of moisture and oil content as a function of time, which were compared to the experimental data. The model could predict the experimental moisture and oil content data with reasonable accuracy. Simulations indicated that most physical processes approach a steady state in the first 20 sec of frying. Thus, in the industry, energy can be saved by reducing frying time from 60 sec to 20 sec. The rate of moisture loss and oil uptake was found to be directly related to the pressure development, evaporation rate and frying temperature. Using fluid distribution, pressure development, temperature distribution and mechanical changes, physical insights about the transport mechanisms during frying were obtained.

Keywords: Hybrid mixture theory, frying of foods, multiscale equations

1. Introduction

Reduction of fat uptake in fried foods such as rice crackers is a difficult task because they are thin and rigid initially, and form a puffy and expanded structure in few seconds. During frying, transport of water, oil and vapors occurs in the food matrix over a hierarchy of spatial scales at a rapid rate. Hybrid mixture theory (HMT) of porous media was used to integrate the transport processes with quality changes at micro and macroscales. Microscale was the scale of interaction of polymers with surrounding water, vapor and oil phases. General transport equations developed using the framework of HMT were solved using the general PDE option of Comsol Multiphysics.

2. Modelling Equations

After combining the generalized Darcy’s law with the mass balance equations, the generalized transport laws can be simplified following (Takhar 2009) in multiphase flow. Various terms of these equations are described by (Maneerote and Takhar 2009).

Water phase:
\[ (1 - \varepsilon_w^o - \varepsilon_v^o - \varepsilon_a^o) \dot{\varepsilon}_w^o - (1 - \varepsilon_w^v - \varepsilon_v^v - \varepsilon_a^v) \]
\[ + \varepsilon_w^v \left( \frac{\varepsilon_w^v}{\mu} \left( F_{ij}^v + \varepsilon_v^v \rho^v \frac{\partial A^v}{\partial E_{KL}} \hat{E}_{KL,i} + \varepsilon_w^v (N^v \hat{e}_w^v)_{ij} \right) \right) ]_j + \varepsilon_w^v (\dot{\varepsilon}_w^o + \dot{\varepsilon}_w^v + \dot{\varepsilon}_w^a) \]
\[ = - \frac{(1 - \varepsilon_w^o - \varepsilon_v^o - \varepsilon_a^o)}{\rho_w} \dot{e}_w \]

Vapor phase:
\[ (1 - \varepsilon_v^o - \varepsilon_v^v - \varepsilon_v^a) \dot{\varepsilon}_v^o - (1 - \varepsilon_v^o - \varepsilon_v^v - \varepsilon_v^a) \]
\[ + \varepsilon_v^v \left( \frac{\varepsilon_v^v}{\mu} \left( F_{ij}^v + \varepsilon_v^v \rho_v \frac{\partial A^v}{\partial E_{KL}} \hat{E}_{KL,i} + \varepsilon_v^v (N^v \hat{e}_v^v)_{ij} \right) \right) ]_j + \varepsilon_v^v (\dot{\varepsilon}_v^o + \dot{\varepsilon}_v^v + \dot{\varepsilon}_v^a) = 0 \]

Oil Phase:
\[ (1 - \varepsilon_w^o - \varepsilon_v^o - \varepsilon_a^o) \dot{\varepsilon}_a^o - (1 - \varepsilon_w^o - \varepsilon_v^o - \varepsilon_a^o) \]
\[ + \varepsilon_v^o \left( \frac{\varepsilon_v^o}{\mu} \left( F_{ij}^o + \varepsilon_v^o \rho_o \frac{\partial A^o}{\partial E_{KL}} \hat{E}_{KL,i} + \varepsilon_v^o (N^o \hat{e}_v^o)_{ij} \right) \right) ]_j \]
\[ = (1 - \varepsilon_v^o - \varepsilon_v^o - \varepsilon_v^a) \dot{\varepsilon}_o^a = \left( \frac{\varepsilon_v^a}{\rho_o} \right) \frac{D \rho_o}{D} \]

3. Results & Discussion
3.1 Validation of average moisture content values

Volume fraction values for water were calculated by numerically solving above equations. The calculated values of volume fraction of water were converted to the average moisture content by integrating over the area of whole specimens at each time step. The model accuracy was verified by calculating the $R^2$ of nonlinear regression and the root mean square error (RMSE). The $R^2$ and RSME values for the average moisture content at various frying temperatures were in the range of 0.94-0.99 and 0.01 to 0.021, respectively. The following figure shows the average moisture content profiles for frying temperatures 473, 493 and 513 K. The high $R^2$ (0.94-0.99) and small RMSE (0.01-0.021) values indicate that the predicted data were in good agreement with the experimental data. When the frying time increased, the average moisture content of deep fried rice crackers decreased as expected. The average moisture content profiles at all frying temperatures were similar to curves commonly observed for drying of foods. The falling rate period was observed in the first 20s. After this, the slope of drying curve decreased. The drying curve finally reached equilibrium and the slope was nearly zero after 30-40 s. The average moisture content of deep fried rice crackers decreased with increase in frying temperature. The slope of the graph during falling rate period for the frying temperature of 513 K was lower than the slopes at 473 and 493 K.

3.2 Water Content Distribution

The two-dimensional profile of volume fraction of water is presented in the following figure. The cross sectional profiles of the rice crackers at various frying times were represented by six subplots. High volume fraction of water (greater than 0.08) is represented as the dark red color and the low volume fraction of water (less than 0.02) is represented as the dark blue color. When the frying time increased, the volume fraction of water decreased. The volume fraction of water significantly reduced within 10 s, and then gradually decreased. After 30 s of frying time, the volume fraction of water slowly decreased and nearly reached the equilibrium at 60 s. For the frying time of 60 s, although the actual volume fraction of water difference was very small, the colors of the figure were distinct. The volume fraction of water indicated the level of water in the rice crackers. A rapid decrease in water at the initial of frying was because of the large temperature difference between the oil and the crackers. A similar conclusion was made by
Halder et al. (2007) for French fries. The surface water in the crackers markedly evaporated, and resulted in the migration of water from the center to the surface. These results probably accounted for the high evaporation rate and the high pressure buildup at the beginning of frying. When the frying time increased, the temperature difference between the oil and the crackers was smaller resulting in the decline in volume fraction of water.

\[ \text{Fig. 2. Water content distribution} \]

### 3.3 Oil Content Distribution

The following figure shows the two-dimensional profile of the volume fraction of oil. The dark blue color represents low volume fraction of oil (less than 0.10) and the dark red color represents high volume fraction of oil (greater than 0.45). The six subplots present the distribution of volume fraction of oil for various frying times. The volume fraction of oil represents the oil uptake during frying. The volume fraction of oil increased with frying time. This result was in accordance with the previous literature. The volume fraction of oil content in rice crackers increased slightly for the first 20 s and gradually increased until 60 s of frying time. This was probably due to a quick pressure buildup in the pore of rice crackers for the first 20 s that prevented oil uptake. The pressure buildup could help to prevent the capillary diffusion of oil in the beginning of frying (Yamsaengsung and Moreira 2002). A rapid increase in pressure led to rapid volume expansion, which resulted in the high amount of pores inside the fried rice crackers. This result was consistent with the results from the experimental data of porosity. The porosity of fried rice crackers significantly increased in the first 20 s of frying and remained constant afterward. The results showed that the crackers were completely expanded in 20 s. The water vapor escape along with expansion of fried rice cracker may be reduced the pressure in the pores of the rice crackers. Moreover, the adsorbed oil probably penetrated to the pore of rice crackers by the capillary diffusion. This absorption may also have reduced the pressure to nearly ambient pressure.

\[ \text{Fig. 3. Oil content distribution} \]

### 4. Conclusions

The hybrid mixture theory based water, oil and vapor transport equations were implemented in Comsol Multiphysics using the general PDE option. Simulations indicated that most physical processes approach steady state in the first 20 sec. Thus, in the industry, energy can be saved
by reducing frying time from 60 sec to 20 sec. The rate of moisture loss and oil uptake was found to be directly related to the pressure development, evaporation rate and frying temperature. Using fluid distribution, pressure development, temperature distribution and mechanical changes, physical insights about the transport mechanisms during frying were obtained.

5. References


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