Thermally Induced-Noise Reduction Using an Electrostatic Force Feedback

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Introduction to MEMS

• MEMS gave versatile sensing solutions
  - Gyroscope
  - Accelerometer
  - Bio-Sensors

• MEMS have various advantages
  - Low cost and high performance
  - Small size
Thermal Noise in MEMS

- Thermal Agitation
  - Caused by temperature fluctuation
  - Inconsiderable in macro-scale
  - Becomes significant in micro-scale

Air Molecules
Thermal Noise in MEMS

- Displacement of a mass-spring oscillator

\[ \frac{1}{2} k \langle x^2 \rangle = \frac{1}{2} K_B T \]  \[\text{[1]}\]

- \( k \) = spring constant
- \( x \) = mean-square displacement
- \( K_B = 1.38 \times 10^{-23} \) J/K (Boltzmann’s constant)
- \( T \) = temperature
Thermal Noise in MEMS

**Example**) At T=300K, a micro cantilever with an effective stiffness of $k=1e^{-3}[N/m]$ will have an expected displacement amplitude $<x>$ about ~2nm.

Not desirable for devices such as AFM which handles molecular scale measurements.
Electrostatic Force Feedback

Previous usages of force feedback

- Extend sensor bandwidth beyond $\omega_0$. [2]
- Nonlinearities in capacitive pickoff minimized [2]
- Decrease spring constant for high performance [2], [3]
Electrostatic Force Feedback

![Diagram of electrostatic force feedback](image)

\[ F_{\text{electrostatic}} = N\varepsilon_0 V^2 \frac{h}{g} \quad [N] \]

\[ x = \frac{\varepsilon_0 h N}{g \cdot k_x} V^2 \quad [m] \]

\[ C = \frac{2N \varepsilon_0 h \cdot l}{g} \]

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( l )</td>
<td>Overlap of the fingers</td>
</tr>
<tr>
<td>( g )</td>
<td>Gap between the fingers</td>
</tr>
<tr>
<td>( w )</td>
<td>Width of a finger</td>
</tr>
<tr>
<td>( h )</td>
<td>Thickness of the device</td>
</tr>
<tr>
<td>( N )</td>
<td>Number of fingers</td>
</tr>
</tbody>
</table>

*Table 1. Important geometric variables for comb drive*
Electrostatic Force Feedback

• Single-ended sensing interface
  – position measurement by applying $V_s$ pulse at capacitive half bridge[2].
  – Capacitive imbalance cause different amount of charge flow[2].
Modeling in COMSOL

• MEMS > 2D-Plane Stress & Electrostatics

\[ F_{\text{noise}} \]

Negative feedback

Positive feedback
Modeling in COMSOL

• Modeling Random Noise

Option > Functions > New > File

Random arrays of numbers were created using MATLAB.

Global Expression > $F_{\text{noise}} = (\text{amplitude}) \times \text{random(t)}$
Modeling in COMSOL

• Feedback Voltage Expression

\[ F_x = F_{\text{Electrostatic}} \]

\[ x \cdot k_x = N \varepsilon_0 V^2 \frac{h}{g} \]

\[ v_{fb1} = \sqrt{\frac{g \cdot k_x}{\varepsilon_0 h N} \frac{d(disp_n)}{dt}} \]

\[ v_{fb2} = \sqrt{\frac{g \cdot k_x}{\varepsilon_0 h N} \frac{d(disp_p)}{dt}} \]
Modeling in COMSOL

• Feedback Voltage Expression

\[ F_x = F_{Electrostatic} \]

\[ x \cdot k_x = N \varepsilon_0 V^2 \frac{h}{g} \]

\[ V_{fb1} = \sqrt{\frac{g \cdot k_x}{\varepsilon_0 h N}} \cdot \frac{d(\text{disp}_1)}{dt} \]

\[ V_{fb2} = \sqrt{\frac{g \cdot k_x}{\varepsilon_0 h N}} \cdot \frac{d(\text{disp}_2)}{dt} \]
Modeling in COMSOL

• Summary of Simulation

1. Apply Noise
2. Obtain disp.  >  Apply Feedback V
Challenges in COMSOL

- Modeling sensing interface with SPICE

Physics > SPICE Circuit Editor

Floating Potential
Conclusion

• Electrostatic force feedback reduces the amplitude of noise induced displacement
• More careful modeling necessary for more significant reduction
  – Randomized noise
  – Realistic Geometry
  – Sensing Interface
References


