NUMERICAL SIMULATION OF EXACT TWO-DIMENSIONAL GOVERNING EQUATIONS FOR INTERNAL CONDENSING FLOWS

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Condensing Flows in Applications

Electronic/Computer Cooling

Space Based Applications

Heat pipes, Rankine Power Cycles, Thermal Management Systems, Design of ISS-based two-phase flow facility, etc.

Research Purpose:
- Facilitate more effective design of “thermal” systems – miniature or not.
- Significantly enhance chance of successful operation of condensers in ground/space based applications.
Overview:

- Problem Formulation
- COMSOL / MATLAB Implementation
- Validation of Computational Results
  - By Other Computational Tools
  - With Experiments
- Differences between Gravity Dominated and Shear Driven Flows
Background Literature

• Available knowledge for exact and approximate model equations for two-phase flows and interface.

• Classical solutions of external condensing flow problems.

• Experimental data and correlations for internal condensing flow problems.

• Level-set methods and its implementation

• Experimental data and correlations for external flow problems.

• Analytical and semi-empirical data and theoretical results for internal condensing flows.

• Theoretical results on dynamic instabilities and turbulence.

Next..
Basic Simulation Strategy
Based on first-principles

- Continuum governing equations
- Interface conditions

(Kinematics, Mass, Momentum, Energy Transfer & Thermodynamic)

**Other conditions**
- Wall conditions
- Conditions at infinity (if any)
- Inlet/outlet conditions
- Initial conditions \((t = 0)\)

**Special features**
- Latent heat released with huge increase in density
- Interface conditions bring in additional non-linearities – they connect the vapor and liquid flows, and also determine its time varying location
COMSOL/MATLAB Based Simulation Tool

Vapor Domain

Liquid Domain

$U, T_{\text{sat}}(p_{\text{in}})$

Outflow

$p_{\text{in}}$

Outflow

Vapor

Interface Condition

$T_{\text{wall}}(x)$ is specified/known

or

Heat flux is specified/known
Data Processing through Equation

Input/output

Data Extraction

Liquid Domain

Vapor Domain

MATLAB

COMSOL

Modules
- Fluid Dynamics
- Heat Transfer
- Deformed Mesh
- Level-Set ???
Current Simulation Capabilities
Annular Internal Condensing Flows

- Boundary value problem (BVP) for steady solutions
- Initial boundary value problem (IBVP) for unsteady solutions
Validation of Computational Results

1. The computational results from the COMSOL / MATLAB tool is compared with the following computational tools:
   ✓ 2-D simulation tool based on SIMPLER algorithm
   ✓ Independently developed 1-D analytical tool

2. The computational results are also compared with the experimental results:
   ✓ Internal condensing flows inside an inclined channel  (Lu & Suryanarayana)
Validation by Comparison with Other Computational Results

Steady Solution Obtained by Different Computational Tools

- Initial Guess - 0.95 Steady Solution
- Initial Guess - 0.9 Steady Solution
- Solution from COMSOL based tool
- 1D solution
- Solution from Fortran based tool

Film Thickness, $\Delta$ (m)

Dimensional distance along the channel, x (m)
Gravity Driven and Shear Driven Flows

Gravity driven and shear driven flows are quite different with regard to

Film Thickness Values Obtained from the Unsteady Simulation

- Initial guess
- Non-dim. time = 7
- Non-dim. time = 15
- Non-dim. time = 17
- Non-dim. time = 19
- 1D steady solution

Distance along the length of channel, x (m)

Film Thickness, $\Delta$ (m)
Comparison of the Comsol Simulation with Lu and Suryanarayana for Run-213

Plot of Film Thickness - R113, Run-213

Comparison of the Comsol Simulation with Lu and Suryanarayana for Run-213
Validation by Comparison with Experiments of Lu and Suryanarayana

<table>
<thead>
<tr>
<th>Run</th>
<th>Fluid</th>
<th>U (m/s)</th>
<th>Delta_T (°C)</th>
<th>Film Thickness Experimental, mm</th>
<th>Film Thickness Computational, mm</th>
<th>Error between Exp and Comp (%)</th>
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<tbody>
<tr>
<td>208</td>
<td>R-113</td>
<td>0.861764</td>
<td>22.28</td>
<td>0.147 0.296 0.344 0.37 0.4</td>
<td>0.226 0.298 0.339 0.394 0.456</td>
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<td>0.106 0.195 0.247 0.345 0.38</td>
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<tr>
<td>206</td>
<td>R-113</td>
<td>1.710475</td>
<td>30.95</td>
<td>0.17 0.28 0.34 0.375 0.412</td>
<td>0.236 0.309 0.350 0.405 0.466</td>
<td>10.7</td>
</tr>
</tbody>
</table>
Annular flow regime is responsible for rejecting most of the heat from a condenser. The associated liquid condensate motion is strongly affected by the orientation of the gravity vector $\bar{g}$ if the duct’s hydraulic diameter $D_H \geq 2\text{mm}$. 

Relevant Results
Gravity Driven and Shear Driven Flows

Gravity driven and shear driven flows are quite different with regard to film thickness, velocity, and temperature profiles.

![Graph showing u-velocity profiles and temperature profiles for gravity driven and shear driven flows.](chart.png)

- Film thickness for horizontal condenser
- Temperature profile for horizontal condenser
- Film thickness for vertical condenser
- Temperature profile for vertical condenser

**Temperature profiles**

- Linear profile at \( x = 5 \)
- Parabolic profile

**U-velocity profiles**

- \( @ x = 20 \)

**Non-dimensional distance along \( x \) direction, non-dimensional \( u \)-velocity, and temperature \( (^\circ \text{C}) \).**
Gravity Driven and Shear Driven Flows

Gravity driven and shear driven flows are quite different with regard to cross-sectional pressure variations as well.

Interfacial Pressure profile for gravity driven and shear driven flows

- **Gravity Driven and Shear Driven Flows**

Pressure difference for flow in a vertical condenser at a modest $\Delta T$

Pressure difference for flow in a horizontal condenser at a modest $\Delta T$
Gravity Driven and Shear Driven Flows

For small inclinations (~ 10°) of the condensing plate, the flow becomes gravity driven in mm-scale condensing flows.

- $\theta$: Tilt Angle (Degrees)
- $\mathbf{g}$: Time-varying gravity vector

Effect of time-varying gravity vector ($\mathbf{g}$) on film thickness

**Graph:**
- Time Varying Tilting of Horizontal Plate
- Time (seconds) vs. Tilt Angle (Degrees)
- $V_1$: Distance along the condenser (Non dimensional)
- $V_2$: Film thickness (Non dimensional)

For small inclinations (~ 10°) of the condensing plate, the flow becomes gravity driven in mm-scale condensing flows.
Summary

• An algorithm for successful and accurate computational simulations of steady and unsteady condensing flows has been presented.

• The results from the computational tool using COMSOL are in good agreement with the 2-D computational code based on SIMPLER Algorithm and a completely independent quasi 1-D tool.

• Relevant results from the reported computational tool developed here are shown to be in agreement with the experimental results for the inclined channel flow experiments.

• Differences between gravity dominated flow and shear driven flows are discussed.
Thank You

Questions?
Vertical Flows for mm-μm Scale Condensers

A condenser’s gravity-sensitivity can be minimized by using suitable array’s of μm-scale ducts. This makes body force effects small relative to shear forces – at a pressure penalty.

\[
F_{\text{body}} > F_{\text{shear}} \quad \text{mm-scale condenser} \\
F_{\text{body}} \ll F_{\text{shear}} \quad \text{μm-scale condenser}
\]
Condensing Flows in μm Scale Ducts are Shear/Pressure Driven

Gravity parameter $G_p \equiv \left( \frac{\rho_2 g x D_h^3}{\mu_2^2} \right)$ is reduced by $D_h \rightarrow \mu$m scale, and $D_h < D_{cr}$, the flow becomes shear driven for a range of gravity values and for a given average inlet speed, $\Delta T$, and working fluid.
Effect of Hydraulic Diameter on the Nature of Flow

As $D_h \rightarrow \mu m$ scale, and $D_h < D_{cr}$, the flow becomes shear driven and gravity-insensitive.

![Graph showing the effect of hydraulic diameter on flow properties.](image-url)

- **Case A**: Large Diameter
- **Case B**: Same as $\delta_{PS}$
- **Case C**: Solution changes very slowly for $D < D_{cr}$
- **Case D**: Film thickness, $\Delta (m)$ vs. Length along axial direction, $x (m)$
Effect of Hydraulic Diameter on the Nature of Flow

As $D_h \rightarrow \mu$m scale, and $D_h < D_{cr}$, the flow becomes shear driven and is accompanied with significant rise in pressure drop.

Ongoing research investigations for condensing flows in $\mu$m-scale ducts will account for:

- Significant $T_s(p^i)$ variation over the flow
- Significant vapor density variation
- Significant surface tension effects
Validation of Computational Results and Experiments

The condensing flow simulation results presented earlier are based on accurate computational simulations that have been quantitatively verified by experiments for gravity driven flows.

Flow Regimes in Internal Condensing Flows

Gravity Driven Flows ($D_h > 1$ mm)
- Mostly Annular
  
  (Rabas et. al. [2000], Narain et. al. [2009] – [2010]*)

Shear Driven Flows
- Annular for:
  - Horizontal mm scale partially condensing flows
- Results are consistent with Cheng et. al. [2005], Garimella et. al. [1999] (Complex Morphology)

(?) ← Horizontal Tube (> mm-scale) → (?)
Sensitivity of Boiling/Evaporating Flows to Gravity Vector

For flow inside boilers/evaporators, the effects of \( \mathbf{g} \) vector changes are expected to be less dramatic as compared to flow inside condensers. This is because “thermal” boundary conditions on the heater surface primarily couple with inlet mass flow rate values - which causes body forces to have a secondary influence on heat transfer rates.

![Diagram of flow boiling](image)

Courtesy: Incropera et.al [Textbook].

The sensitivity of flow boilers need to be ascertained (**research is needed**). We do not know what flow boiling information exists with regard to g-sensitivity of Fairchild Corporation’s existing aircraft designs. Our forthcoming boiler experiments require that, for air force needs, the boiler be placed on a **suitable shaker**.
Theoretical and experimental results on “Elliptic-sensitivity” are presented for condensers.

Analogous experimental results for boilers are expected within a year.
Shear/Pressure driven condensing (boiling?) flows exhibit a key phenomenon due to fundamentally different behavior compared to gravity driven flows. This is marked on our transition map for annular internal condensing flows.
“Elliptic – Sensitivity” for Shear Driven Internal Condensing Flows  
(Consider Partially Condensing Annular/Stratified Flows)

In the above thought experiment, one asks whether the exit condensate flow rate ($\dot{M}_{L-e}$) (or natural pressure difference $\Delta p$) can be changed to achieve multiple quasi-steady solutions (not necessarily annular/stratified). In other words: do these flows exhibit “elliptic-sensitivity” (i.e. do these flows listen to both upstream and downstream conditions) ?

- Yes! Because net mean energy into the control volume can be changed by a change in the interface energy transfer (associated with interface location and mass transfer).

Clearly, the above different $\Delta p$ impositions are impossible for single-phase flows or adiabatic two-phase flows (with zero interfacial mass transfer) because, the information only travels downstream (i.e. they are parabolic flows), and energy flow across the interface being zero.
Basic Results on the Special Nature of Condensing Flows

- For any internal condensing flow (shear or gravity driven), there is a unique steady (termed “natural”) annular/stratified (or “film” condensation) solution/realization which can be realized when the set up allows the flow to seek its own exit condition.

- However one can “actively” impose different steady or quasi-steady exit conditions - other than the “natural” one - for purely shear driven or “mixed” flows. This typically leads to other time dependent or quasi-steady solutions which may cause the flow regime boundaries (from annular stratified to non-annular (plug, slug, etc.) flows) to shift.
Test-Section and Schematic of the Observed Flow

Annular/Stratified flows → Plug/Slug flows → Bubbly flows

Vapor exit (closed for full condensation)

L = 1 m

HFX – 1, HFX – 2: These are heat flux meters which have thermo-electric coolers underneath them.
Experimental Proof of Elliptic-Sensitivity

Imposition of different $\Delta P$  
$\Delta P_{\text{Na}} \neq \Delta P_{\text{imposed}}$

Mean exit pressure: fixed

Mean inlet mass flow rate: fixed

$\Delta P_{\text{Na}}$
Experimental Proof of Elliptic-Sensitivity

For this representative case, the 90 Pa change in Δp results in approximately 38 % enhancement in the average heat flux.
Experimental Proof of Elliptic-Sensitivity

The increase in heat transfer rates have to negotiate the resistance which leads to the thermal transients.

For $x > 60$ cm, condensing-surface temperatures were significantly reduced by HFX-2 to shrink the non-annular flow regimes and to achieve all liquid flow at $x = L$.
Theoretical/Computational Proof of Elliptic-Sensitivity

**Input:** Imposition of different $M_{L-e} (t)$

![Graph showing non-dimensional liquid exit flow rate vs non-dimensional time for different control types](graph.png)

- Continuous 'Off-Natural' Control
- On-Off Control for an 'Off-Natural' Mean
- Continuous Control at 'Natural' Exit Flow Condition

Mean Value of the 'On-Off' Control
Strictly steady solution
Quasi-steady/periodic imposition of fluctuations

$M_{L-e-c-1} (t)$  $M_{L-e-c2} (t)$  $M_{L-e-Natural}$

**Equations:**

$\Delta(x,t)$

$P_1 \rightarrow M_{e-c1}(t)$  $P_2 \rightarrow M_{e-c2}(t)$  $\rightarrow M_{e-Natural}$

Condensing Surface
Liquid
Vapor

$\frac{\partial}{\partial t} \frac{\partial}{\partial x} \left( \frac{M_e}{P} \right)$

$\frac{1}{g}$
Control through Liquid Mass Flow Rate at the Exit

For all times annular/stratified solutions exist for these “special” controls at or near “natural” value.

Solution becomes non-annular at time $t > t^*$ after certain time $t^*$ ($t > t^*$). No annular stratified solution exists for all “off-natural” constant steady controls. This is further substantiated by the instability result for a constant steady control case with mean at an “off-natural” value.

At all times $t > 0$

At time $t < t^*$
Dynamic Stability at “Natural” and Instability for Continuous “Off-Natural” Control

Steady Imposition at “Natural” is Stable

“Off-Natural” Steady Imposition is Unstable

But “Off-Natural” Quasi-Steady Imposition is Robust
Energy Response to Exit Condition Imposition

**Consider:** Non-Dimensional Net Mechanical Energy into the Condenser (Partial Condensation) versus time

- For constant steady control of exit liquid mass flow rate at “off-natural” value, energy keeps piling with a non-zero positive slope or draining inside the domain. This leads, eventually, to a situation where annular/stratified solutions do not exist after a certain transition time. These annular/stratified flows are unstable and only their transition behavior for negligible to non-zero initial disturbances can be computationally studied with the help of the current simulation technique.

- However, for on-off control with the mean of the control near the “natural” value, mean energy in the condenser also settles down to a steady value near the steady “natural” value and, therefore, nearby quasi-steady annular/stratified solutions exist and thus, flow regime transition is avoided.

### Controllability Through Liquid Exit Mass Flow Rate

- **Constant Steady Control at ‘Natural’ Exit Condition**
- **Constant Steady Control at an ‘Off-Natural’ Mean**
- **On-Off Control at an ‘Off-Natural’ Mean**

Mean steady “off-natural” value is achieved for an on-off control in the vicinity of the “natural.”

Typically negative slopes are associated with the constant steady “off-natural” control.
Energy Response to Exit Condition Imposition

**Consider**: Mean Non-Dimensional Net Mechanical Energy into the Condenser (Partial Condensation) for Different Quasi-Steady Realizations

Annular flows realized through “on-off” control in this range correspond to the limited steady energy band associated with this control.

For “periodic” steady-in-the-mean realizations in the vicinity of strictly steady “natural” realization indicated above– nearby quasi-steady solutions exist. Therefore, in the presence of fluctuations, PID control of both the mean inlet and the exit pressures become feasible.
Energy Response to Exit Condition Imposition

Consider: Mean Overall Interface Energy Transfer Rates (Non-Dimensional) for Different Quasi-Steady Realizations

Assembled Results on Interfacial Energy Transfer for "Natural" and Nearby Quasi-steady Prescriptions

Mean Non-Dimensional Liquid Mass Flow Rate at the Exit
EFFECT OF ELLIPTIC-SENSITIVITY ON SYSTEM PERFORMANCE

(ONGOING AND FUTURE RESEARCH)
Implications of “Elliptic-Sensitivity” on System Level Repeatability

For the same steady $Q_B$, $M$, $T_{source}$, and $T_{sink}$; and a transient load history shown below, there could be significant drifts in boiler temperature for a given load history due to elliptic-sensitivity associated with the two-phase components.

That is, performance of boiler at Time I may not be same as that at Time I”. 

![Diagram of boiler system showing temperature relationships and load history](image-url)
Research Needs

• We need to learn what aspects of our facility can be altered/upgraded to address issues of interest to ongoing Air Force-Fairchild Corporation’s CRADA.

• We would like to collaborate and learn from you. We can do this by doing experiments and simulations that are not covered by NSF (3 year grant on μm-scale condensing flows) and NASA (1 year grant on flow boiling) grants. The possibilities are:
  i. Do a μm-scale tubular boiling experimental research of Air Force’s interest.
  ii. Collaborate with Air Force on putting our μm-scale boilers/condensers on a suitable shaker that model different g-force history segments of your interest. For this, we may need a miniaturization of our facility as well as change in our working fluid (from FC-72 to fluids of your interest). The new equipment can be developed at AFRL or MTU. However suitable shaker experiments can be performed by additions to the existing MTU facility.
  iii. Provide simulation support for annular regime condensing/boiling flows under different g-force histories (e.g. on shakers, aircraft g-force history, etc.)
  iv. Learn your planned system and flow control details to see how “elliptic/parabolic sensitivity” issues discovered (and being developed) by us for flow condensation and flow boiling may be of assistance to you.

★
Conclusions

1. Novel “transition maps” for steady annular/stratified condensing flows yield a proper subdivision of the parameter space into gravity, shear, and mixed driven flow zones. This is of help in a-priori estimation of effects of changes in steady gravity levels.

2. For mm-scale ducts, the steady flow computational results as obtained from the 1-D/2-D solution techniques have been validated by comparisons with vertical tube experiments. Similar computational results and associated horizontal channel condensing flow experimental results are being synthesized.

3. “Elliptic-sensitivity” results for condensing flows (results for boiling flows are expected) were established both theoretically and experimentally. Its significance for controlling thermal transients and ensuring system level repeatability was discussed.
Thank You

Questions ?
Experiments Using a Shaker to Assess Gravity-Sensitivity

Can the response for any duration \([t_1, t_2]\) in an actual flight trajectory be assessed by mounting the boiler/condenser on a shaker with a periodic acceleration as shown above?

Sample g-force vector history for a particular aircraft maneuver in a vertical plane

Representative \(\bar{g}(t)\) profile for experiments

Small enough duration to minimize impact on flow

Can the response for any duration \([t_1, t_2]\) in an actual flight trajectory be assessed by mounting the boiler/condenser on a shaker with a periodic acceleration as shown above?
Available Knowledge/Literature

Fundamental Laminar/Laminar Solutions:

External Flows

- Stagnant Vapor
  - Nusselt Problem [1914], Narain et. al. [2007]

Internal Flows

- 1g or 0g
  - Koh Problem [1961], Narain et. al. [2010]*
  - (Narain et. al. [2004], [2009], [2010]*)

Experimental Investigations:

- Correlation for average heat transfer coefficients: (Cavallini et. al. [1974], Shah et. al.[1979], etc.)
- Flow regime visualization (Garimella et. al. [1999], Cheng et. al. [2005]), Flow regime maps (Carey [1992])

Gravity Driven Flows (Dh > 1 mm)
- Mostly Annular
  - (Rabas et. al. [2000], Narain et. al. [2009] – [2010]*)

Shear Driven Flows
- Annular for:
  - Horizontal mm scale partially condensing flows
  - Small μm scale channel and cylinder with “surface tension effects” under “self-selected” exit conditions (Narain et. al. [2010]*).
- More commonly: Complex Morphology (Cheng et. al. [2005], Garimela et. al. [1999]), Narain et al. [2010]*

(?) ← Horizontal Tube (> mm-scale) → (?)
Internal Flows

Gravity Driven Flows ($D_h > 1 \text{ mm}$)
Mostly Annular

(Narain et. al. [2009] – [2010]*)
Shear Driven Flows

Annular Flow
Research Tools that are Employed for Reported Results and Planned Research

Computational Simulation Tool

Experimental Flow Loop Facility

Experiments
Fluid: FC72
Vertical Tube: $D_h = 6 \text{ mm}$
$2 \leq G \leq 90 \text{ kg/m}^2\text{-s}$
Horizontal Channel: $D_h = 2 \text{ mm}$
$2 \leq G \leq 200 \text{ kg/m}^2\text{-s}$

Newly Invented Film Thickness Sensor  (Narain et. al., JHT, 2010)
First Principles Underlying Flow Physics and Computational Problem

- **Continuum governing equations** (Mass, momentum, and energy for each differential element in the interior of the two phases)
- **Interface conditions** (on the unknown interface these are restrictions imposed by: kinematics, mass transfer, momentum transfer, energy transfer, and thermodynamics)

Other conditions
- Wall conditions
- Conditions at infinity (if any)
- Initial conditions \((t = 0)\)
- Inlet conditions
- Exit conditions (need ?)

Special features
- Sharp interface
- Single-phase solutions interact through interface conditions
- Interface condition is used for interface tracking
  - Height function with adaptive grid (current)
  - Level-set function (planned)
Experimental Facility for Internal Condensing Flows

- Evaporator
- Inlet Flow Valve
- Coriolis Flow Meter
- Test Section
- L/V Separator
- Rotameter
- Auxiliary Condenser
- Data Acquisition
- Displacement Pump 1
- Displacement Pump 2
- Vacuum Pump
- Water Pump
- 15
Experimental Test-Section

Vertical Tube

Horizontal Channel

(a) (b)

- Nyloa entrance and exit sections
- Refrigerant tube wall
- Developed liquid film
- Flowing vapor
Combined Experimental Facility
Comparison of Results Obtained by 1-D and 2-D Solution Techniques for Annular/Stratified Flows

Gravity Driven Flow in mm Scale Vertical Ducts

Shear Driven Flow in 0g, Horizontal, and μm Scale Ducts

The 2-D and 1-D prediction for other flow variables (interfacial velocity, pressure, etc.) exhibit similar good agreements for different flow conditions and tube geometries as well. Within their own regimes, they also agree with experiments.

Both 1g and 0g flows are stable. Note: (i) gravity driven smooth flows become wavy for Re₈ > 30, but they remain annular/stratified. (ii) Shear driven & 0g flows – though stable (as shown) are not always experimentally realized – except under “controlled” conditions.
COMPUTATIONAL RESULTS ON COMPARISONS BETWEEN GRAVITY DRIVEN FLOWS AND SHEAR DRIVEN FLOWS
Transition Between Gravity Driven and Shear Driven Flows

Parameters affecting the flows: \( \{x, \Re_{in}, G_p \equiv \operatorname{Fr}^{-1}_x \Re_{in}^2, \operatorname{Fr}^{-1}_y = 0, \Ja/\Pr_1, \rho_2/\rho_1, \mu_2/\mu_1\} \)

Method of Cooling: \( T_w(x) = \text{Constant} \)

\[ \begin{align*}
\partial_1 \Sigma_1: & \ x^* \approx 0 \\
\partial_2 \Sigma_1: & \ x^* \approx x_{0.7} \\
\partial_2 \Sigma_2: & \ x^{**} \approx x_{0.7} \\
\partial_1 \Sigma_2: & \ x^{**} \approx 0
\end{align*} \]

Transition map in \( \{x, \Re_{in}, \operatorname{Fr}^{-1}_x\} \) space for chosen \( \{\Ja/\Pr_1, \rho_2/\rho_1, \mu_2/\mu_1\} = \{0.004, 0.0148, 0.0241\} \)

\[ G_p \equiv \operatorname{Fr}^{-1}_x \Re_{in}^2 \equiv \left( \rho_2^2 g_x D_h^3 \right) / \mu_2^2 \]

(transition within 4% of P.S.) (within 4% of Nusselt)
Transition Between Gravity Driven and Shear Driven Flows

In the $Re_{in}$ and $G_p$ plane for a constant $Ja/Pr_1$, $\rho_2/\rho_1$, $\mu_2/\mu_1$
Projection of the Transition Map in $Re_{in}$-$G_p$ Plane and Correlations for Gravity Driven and Shear Driven Flows

For 0g flows,

$$\delta_{ps}(x) = \frac{0.7487^* x^{0.35^*} (Ja_1/Pr_1)^{0.3611^*} (\rho_2/\rho_1)^{0.2380}}{Re_{in}^{0.3529^*} (\mu_2/\mu_1)^{0.5947^*}}$$

$$x_{0.75} = \frac{2.69^* Re_{in}^{0.1826^*} (\rho_2/\rho_1)^{1.1695^*} (\mu_2/\mu_1)^{0.1085}}{(Ja_1/Pr_1)^{0.9011^*} (Fr_x^{-1})^{0.5334^*}}$$

For gravity driven flows,

$$\delta_{Nu}(x) = \frac{1}{Y_c} \left[ \frac{4 \cdot k_1 \cdot \mu_1 \cdot \Delta T \cdot x}{g \cdot \rho_1 \cdot (\rho_1 - \rho_2) \cdot \bar{h}_{fg}} \right]^{1/4}$$

$$= \left[ 4 \cdot \frac{(Ja/Pr_1)}{G_p} \cdot (x/G_p) \right]^{1/4}$$

0g and other correlations (see paper) are for parameter space given by the following:

$$0 \leq x \leq x_A < x_{FC} \text{ or } 0 \leq x \leq x_{0.75}$$

$$900 \leq Re_{in} \leq 22000$$

$$0.0036 \leq Ja/Pr_1 \leq 0.0212$$

$$3.2E-4 \leq \rho_2/\rho_1 \leq 0.03$$

$$0.0113 \leq \mu_2/\mu_1 \leq 0.06$$

$$0.007 \leq Fr_x^{-1} \leq 0.01$$

$$Nu_x = (h_x \cdot L_c)/k_1 = 1/\delta$$

$$q^{''}(x) = h_x \cdot \Delta T$$

$G_p \equiv Fr_x^{-1} \cdot Re_{in}^2 \equiv (\rho_2^2 g_x D_h^3) / \mu_2^2$
EXPERIMENTAL VERIFICATIONS FOR GRAVITY DRIVEN FLOWS
Experimental Data Obtained for Fully Condensing Flows

Range of operating conditions and properties for the experimental data

Gravity Parameter

\( \frac{Ja_1}{Pr_1} \)

\( \frac{\mu_2}{\mu_1} \)

\( \frac{\rho_2}{\rho_1} \)

\( \text{Re}_{in} \)

\( x \times 10^4 \)

\( 10^8 \)
Comparisons Between Theory and Experiments for Partially Condensing Flows (Annular/Stratified Regime)

Given: $\dot{M}_{\text{in}}$

$\Delta T$

is predicted and measured

where: $Z_e = \frac{\dot{M}_{v-e}}{\dot{M}_{\text{in}}}$

<table>
<thead>
<tr>
<th>Run No.</th>
<th>$\dot{M}_{\text{in}}$ (g/s)</th>
<th>$\dot{M}_v$ (g/s)</th>
<th>$Z_e$</th>
<th>$Z_e$ comp 2-D</th>
<th>$Z_e$ comp 1-D</th>
<th>$\bar{T}_w$ (K)</th>
<th>$T_{\text{sat}}$ (K)</th>
<th>$\Delta T$ (K)</th>
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**Percentage agreement within ± 2%**

**Percentage agreement within ± 3%**

Annular Flow Regime Verification Video
Comparisons Between Theory and Experiments for Fully Condensing Flows (Annular/Stratified Regime)

Given: \( \dot{M}_{in} \) and \( \Delta T \) \( x_{FC} \) is predicted and measured
\( \Delta p \) is predicted and measured

There is a good agreement between \( \Delta p = p_{in} - p_{exit} \) obtained both from experiments and predicted by quasi 1-D computational theory

<table>
<thead>
<tr>
<th>Run No.</th>
<th>( \dot{M}_{in} ) (g/s)</th>
<th>( T_{sat} ) (°C)</th>
<th>( \Delta T ) (°C)</th>
<th>( p_{in} ) (kPa)</th>
<th>( p_{xP-3} ) (kPa)</th>
<th>( p_{xP-6} ) (kPa)</th>
<th>( p_{exit} ) (kPa)</th>
<th>( \Delta p ) (kPa)</th>
<th>( \Delta p_{comp} ) (kPa)</th>
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<td>187.15</td>
<td>189.26</td>
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<td>183.82</td>
<td>184.22</td>
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</table>

The agreement is within ± 12%

For shear driven cases, more detailed comparisons between theory and experiments is expected from a better instrumented horizontal rectangular test-section (forthcoming paper).
Comparisons Between Theory (1-D – No Waves) and Experiments for Fully and Partially Condensing Flows

Better agreement with simulations with waves and turbulent effects are possible
Experimental Result Showing the Deviation from Laminar/Laminar Flow and Onset of Turbulence Near the Interface

Discrepancy vs. $\text{Re}_{\text{in}}/(\text{Ja}/\text{Pr})^{0.458}$

- 15%
- 30%
- >30%

Re$_{\text{in}}/(\text{Ja}/\text{Pr})^{0.458}$
Sample Comparisons of Experiments with Various Correlations for Partially Condensing Flows

Comparison of Heat Transfer Values from Different Correlations

- $h_L$ (Simu)
- Nusselt
- Shah
- Dobson-Chato
- Cavallini

0% ±15 % ±30 %
**Recommendation: Use Physics Based Sub-Categories**

Parameters affecting the flows: \( \{x, Re_{in}, G_p \equiv ((\rho_2^2 g_x D_h^3) / \mu_2^2), Ja/Pr_1, \rho_2/\rho_1, \mu_2/\mu_1\} \)

Experimental correlations often replace: \( \{x, Ja/Pr_1, Re_{in}\} \) by \( Re_\delta \) and distance \( x \) by local value of vapor quality \( X \) or \( Z \)

Effectively, for common refrigerants, the parameters \( (Re_{in}, G_p, Re_\delta) \) impose the following restrictions:

- **Small:** if \( Re_{in} < Re_{cr}(x, G_p, Ja/Pr_1) \approx 50,000 \rightarrow \) Laminar Vapor model - OK

- **Large:** if \( Re_{in} > Re_{cr}(x, G_p, Ja/Pr_1) \approx 50,000 \rightarrow \) Vapor Turbulence becomes important

- **Small:** if \( G_p \) is small (?) (\( \mu m \)-scale or \( g_x = 0 \)) \( \rightarrow \) Shear Driven Flows

- **Large:** if \( G_p \) is large (?) (\( mm \)-scale or moderate \( g_x \)) \( \rightarrow \) Gravity Driven Flows

- **Small:** if \( Re_\delta < Re_{\delta cr}(x, G_p, Ja/Pr_1) \approx 1,000 \rightarrow \) Laminar Condensate

- **Large:** if \( Re_\delta > Re_{\delta cr}(x, G_p, Ja/Pr_1) \approx 1,000 \rightarrow \) Turbulent Condensate
## Annular Flow Correlations Practices

Method of Cooling: $T_w(x) = \text{Constant}$

<table>
<thead>
<tr>
<th>Correlations</th>
<th>Low $G_p$</th>
<th>Moderate $G_p$</th>
<th>High $G_p$</th>
<th>Low $Re_\delta$</th>
<th>High $Re_\delta$</th>
<th>Low $Re_{in}$</th>
<th>High $Re_{in}$</th>
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<td>Dobson &amp; Chato [1998]</td>
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<td>Azer et.al. [1971]</td>
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FUNDAMENTAL RESULTS ON EXIT-CONDITION SENSITIVITY AS A PART OF BOUNDARY DATA, FLOW CONTROLLABILITY, AND FLOW REALIZABILITY
Assume Lam/Lam Flows & Look for Annular Flows
Transition Between Gravity Driven and Shear Driven Flows

Parameters affecting the flows: \{x, Re_{in}, G_p \equiv ((\rho_2^2g_x D_h^3) / \mu_2^2), Fr_{in} \equiv 0, Ja/Pr_1, \rho_2/\rho_1, \mu_2/\mu_1\}

Method of Cooling: \(T_w(x) = \text{constant}\)

The above maps takes the goals of Chen, Gerner, and Tien [1986] significantly forward.
Exit Condition Issue for Shear Driven Internal Condensing Flows (Consider Partially Condensing Annular/Stratified Flows)

In the above thought experiment, one asks whether the exit condensate flow rate ($\dot{M}_{L-e}$) (or equivalent exit pressure) can be used to “control” the flow and achieve multiple quasi-steady solutions (not necessarily annular/stratified). In other words: are these flows “elliptic” (i.e. do these flows listen to both upstream and downstream conditions) ?

- **Yes!**

Clearly, the above “control” is impossible for single-phase flows or adiabatic two-phase flows (with zero interfacial mass transfer) because, the information only travels downstream (i.e. they are parabolic flows).

**Related Issues/Questions:**
- What is the nature of the steady governing equations? Are they parabolic (as in single-phase or air-water flows)?
- Are there significant differences between gravity-driven and shear-driven flows?
Summary of Results

The basic result on “ellipticity” is:

- Requires “exit” boundary conditions in general and responds to them.
- But, in the absence of external constraints, “parabolic” boundary conditions (i.e. inlet and wall boundary conditions) suffice for determining the “natural” unconstrained steady solution – not just the annular/stratified type but inclusive of other regimes (plug/slug, bubbly, etc.)

Experimental proof for stable and repeatable “natural” partially condensing shear driven flows
Experimental proof for stable and repeatable “natural” fully condensing shear driven flows

- Imposition of exit conditions- allowed for shear driven flows – changes the liquid/vapor morphology or interface locations and hence significantly changes heat transfer coefficient (or thermal resistance $R_{\text{condensation}}$ for condensing flows). This fact is being experimentally proven.
Differences between Shear Driven Flows in 0g Channel and Horizontal Channel

Unlike 0g flows, at these downstream locations no "natural" exit conditions exist for maintaining annular/stratified flows.

Reasons: See intersecting characteristics on the next slide

Annular/Stratified flow is not possible after $x > x^*$ (Ranjeeth et. al, 2010)
Shear Driven Flow in 0g Channel vs. horizontal Channel

Study of Characteristics for flow in a 0g and horizontal channel

Annular/Stratified flow is not possible after $x > x^*$ (Ranjeeth et. al, 2010)
Shear Driven Flows in a Horizontal Channel

Partial Condensation

Channel Test Section
Inlet and Wall Conditions

- Temperature Difference
- Inlet Mass Flow Rate
- Inlet Absolute Pressure

* Measured at 10 cm downstream test section.
** Temperature difference between calculated $T_{sat}(p_{in})$ and $T_{wall}$ average.
Shear Driven Flows in a Horizontal Channel

Partial Condensation
Channel Test Section
Output Data

Exit Mass Flow Rates (g/s)

Pressure Difference* (kPa)

* Pressure difference between measuring points located at 10 cm and 90 cm downstream test section.
Comparisons Between Theory and Experiments for Fully Condensing Flows (Annular/Stratified Regime)

For a fully condensing flow (with $x_{FC} < L$), $\Delta p \equiv p_{in} - p_{exit}$ is obtained both from experiments and quasi 1-D computational theory. The agreement is within ± 12%.

<table>
<thead>
<tr>
<th>Run No.</th>
<th>$M_{in}$ (g/s)</th>
<th>$\bar{T}_w$ (°C)</th>
<th>$T_{sat}$ (°C)</th>
<th>$\Delta T$ (°C)</th>
<th>$p_{in}$ (kPa)</th>
<th>$p_{xP-3}$ (kPa)</th>
<th>$p_{xP-6}$ (kPa)</th>
<th>$p_{exit}$ (kPa)</th>
<th>$\Delta p$ (kPa)</th>
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<td>-8.51</td>
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More detailed comparisons between theory and experiments is expected from a better instrumented horizontal rectangular test-section (forthcoming).
Conclusions

1. The novel proposed “transition maps” for annular/stratified flows show a proper subdivision of the parameter space into gravity, shear, and mixed flow zones.

2. For mm-scale range, the results – for both gravity and shear driven flows – as obtained from the 1-D solution technique were validated by successful comparisons with 2-D results as well as relevant experimental results.

3. The unique annular/stratified steady solutions define an exit condition which is termed as “natural.” For shear driven flows, unless “natural” exit condition is specified or is accessible, there could be other quasi-steady/unsteady realization of the governing unsteady equations because of inherent exit condition sensitivity. This leads to more complex non-annular flow morphologies.
Generalized Summary

Steady Governing Equations: Are They Parabolic or Elliptic?

Thought Experiment:
The Steady Governing Equations are Neither Elliptic Nor Parabolic

The quasi 1-D/ full 2-D code results indicate that the steady gravity driven flows behave (in most situations) almost like a parabolic flow as it has a strong attractor even in the absence of exit condition specification.

Nature of Steady Solutions and Phase Portrait Diagram
However shear driven (0g or horizontal) internal condensing steady flows behave somewhat like “elliptic” problems as the steady solutions have weak attractors and can be controlled by imposition of exit conditions.
Sensor Principle

An identifiable part of the amount of “green” fluorescent light collected from the fluorescent dopant depends only on instantaneous thickness “δ” for a given concentration of the dopant.

Suitable filtering of noise light associated with light source

Suitable Long Pass Filtering

Fluorescent Signal Detector (Photomultiplier Tube)
First Principles Underlying Flow Physics and Computational Problem

- **Continuum governing equations** (Mass, momentum, and energy for each differential element in the interior of the two phases)

- **Interface conditions** (on the unknown interface these are restrictions imposed by: kinematics, mass transfer, momentum transfer, energy transfer, and thermodynamics)

Other conditions
- Wall conditions
- Conditions at infinity (if any)
- Initial conditions \((t = 0)\)
- Inlet conditions
- Exit conditions (need ?)

Special features
- Sharp interface
- Single-phase solutions interact through interface conditions
- Interface condition is used for interface tracking
  - Height function with adaptive grid (current)
  - Level-set function (planned)
Comparison of Results Obtained by 1-D and 2-D Solution Techniques for Annular/Stratified Flows

Gravity Driven Flow in mm Scale Vertical Ducts
Shear Driven Flow in 0g, Horizontal, and μm Scale Ducts

The 2-D and 1-D prediction for other flow variables (interfacial velocity, pressure, etc.) exhibit similar good agreements for different flow conditions and tube geometries as well. Within their own regimes, they also agree with experiments.

Both 1g and 0g flows are stable. Note: (i) gravity driven smooth flows become wavy for Re₆ > 30, but they remain annular/stratified. (ii) Shear driven & 0g flows – though stable (as shown) are not always experimentally realized – except under “controlled” conditions.
Computational Approach

Adaptive computational grids
Computational Approach

Iterative solution strategy

At discrete number of spatial locations, guess \{\delta, u_{SL}^i, v_{SL}^i, \theta_{SL}^i, u_V^i, v_V^i, \theta_V^i\} for the steady problem at \(t = 0\) and, for the unsteady problem (incompressible vapor and unspecified exit condition) at \(t > 0\), for the). Adjust these \textbf{seven} guess functions: \{\delta, u_{SL}^i, v_{SL}^i, \theta_{SL}^i, u_V^i, v_V^i, \theta_V^i\} with the help of \textbf{seven} interface conditions. The following steps implement this philosophy by separate single-phase (liquid and vapor domain) calculations with a “sharp interface.”
After fixing \{u_{SL}^i, v_{SL}^i, \theta_{SL}^i\} on shifted interface \(\delta_{\text{shifted}}\), solve liquid domain under shifted interface by a finite-volume (SIMPLER) or a finite-element method. The \{u_{SL}^i, v_{SL}^i, \theta_{SL}^i\} are adjusted to satisfy tangential stress (shear), normal stress (pressure), and saturation temperature conditions at the interface.
After fixing \( \{u^i_V, v^i_V, \theta^i_V\} \) on interface \( \delta \), solve vapor domain above interface by the same \textit{finite-volume} method (SIMPLER). UPDATE the guesses for \( u^i_V, v^i_V, \) and \( \theta^i_V \) with the help of: continuity of tangential velocity, interfacial mass flux equality \( \dot{m}_{VK} = \dot{m}_{Energy} \), and saturation temperature conditions at the interface.
Computational Approach

However Popular Level-Set Methods Use

\[ \dot{\text{m}}_{L/K} = \dot{\text{m}}_{\text{Energy}} \quad \rightarrow \quad \frac{\partial \phi}{\partial t} + \vec{V} \cdot \nabla \phi \equiv 0 \]

where, \( \phi (x,t) = 0 \) locates the interface with

\[ \vec{V} \equiv \vec{v}_1 - (k_1 \cdot \nabla T_1)^{\text{extended}} - k_2 \cdot \nabla T_2^{\text{extended}} \cdot (1/\rho_1) \cdot (1/h_{fg}) \]

with subscript I = 1 is for liquid and I = 2 is for vapor

In the new COMSOL/MATLAB based approach, we propose to retain our current approach in principle but use the above level-set equation for locating the interface through \( \phi = 0 \). This will allow investigation of flow regime transitions from annular/stratified flows to plug/slug flows.
Computational Approach

Iterative solution strategy (contd.)

Our Current Practice is to Update δ (by tracking the interface) on an adaptive Eulerian Grid which remains fixed over a time interval [t, t+Δt] of interest.

Current method uses \( \phi = y - \Delta(\chi, t) = 0 \) and “tracks” the interface through the reduced form of \( \hat{m}_{LK} = \hat{m}_{Energy} \) given as:

\[
\frac{\partial \delta}{\partial t} + \bar{u}(x,t) \frac{\partial \delta}{\partial x} = \bar{v}(x,t) \\
\delta(0,t) = 0 \\
\delta(x,0) = \delta_{steady}(x) \text{ or other prescriptions}
\]
Alternative Theory/Computational Results from a Quasi 1-D Semi-Analytical Approach

- The method uses exact analytical solutions of the underlying 2-D governing equations under “thin film” approximation. Only the vapor phase momentum and mass balances employ one-dimensional governing equations with an assumed vapor profile. Hence this method is called “Quasi 1-D.”

For the unknown, \( y(x) = [\delta(x), u_f(x), \pi(x), d\pi/dx (x) \equiv \zeta(x)]^T \), the governing equation

\[
\frac{dy}{dx} = g(y)
\]

is to be solved, subject to the condition

\( y(0) = [\delta(0), u_f(0), \pi(0), \zeta(0)]^T \) or \( y(\varepsilon) = [\delta(\varepsilon), u_f(\varepsilon), \pi(\varepsilon), \zeta(\varepsilon)]^T \) for \( x > \varepsilon \)

Salient Features:
- The problem is singular
- Problem is neither parabolic (because of the presence of \( \zeta(0^+) \) in the \( y(x) \)) – nor clearly elliptic (since explicitly defined values of \( \zeta(0^+) \) is not admissible.)
Strictly steady solution in 1g behaves like a “parabolic” solution as it does not need an exit condition specification.
Nature of Steady Solutions and Phase Portrait Diagram

Strictly steady solution in 0g behaves like an “elliptic” problem with “neutral” to “unstable” steady solution that can take different exit conditions.
Alternative Theory/Computational Results from a Quasi 1-D Semi-Analytical Approach

- The method uses exact analytical solutions of the underlying 2-D governing equations under “thin film” approximation. Only the vapor phase momentum and mass balances employ one-dimensional governing equations with an assumed vapor profile. Hence this method is called “Quasi 1-D.”

For the unknown, \( y(x) = [\delta(x), u_f(x), \pi(x), d\pi/dx (x) \equiv \zeta(x)]^T \), the governing equation
\[
\frac{dy}{dx} = g(y)
\]
is to be solved, subject to the condition
\[
\begin{align*}
y(0) &= [\delta(0), u_f(0), \pi(0), \zeta(0)]^T \\
y(\epsilon) &= [\delta(\epsilon), u_f(\epsilon), \pi(\epsilon), \zeta(\epsilon)]^T
\end{align*}
\]for \( x > \epsilon \)

Salient Features:
- The problem is singular
- Problem is neither parabolic (because of the presence of \( \zeta(0^+) \) in the \( y(x) \)) – nor clearly elliptic (since explicitly defined values of \( \zeta(0^+) \) is not admissible.)
Transition Between Gravity Driven and Shear Driven Flows

Parameters affecting the flows: \( \{x, \text{Re}_{in}, \text{Fr}_{x}^{-1}, \text{Fr}_{y}^{-1} = 0, \text{Ja/Pr}_1, \rho_2/\rho_1, \mu_2/\mu_1\} \)
Transition Between Gravity Driven and Shear Driven Flows

In the $Re_{in}$ and $Fr^{-1}$ for a constant $Ja/Pr$, $\rho_2/\rho_1$, $\mu_2/\mu_1$.
Transition Between Gravity Driven and Shear Driven Flows

For 0g flows,

\[ \delta_{ps}(x) = \frac{0.7487 \times x^{0.35} \times (Ja_1/Pr_1)^{0.3611} \times (\rho_2/\rho_1)^{0.2380}}{Re_{in} \times (\mu_2/\mu_1)^{0.5947}} \]

\[ 0.75x_{FC} = \frac{0.0447 \times Re_{in} \times (\rho_2/\rho_1)^{0.43} \times (\mu_2/\mu_1)^{0.49}}{(Ja_1/Pr_1)^{0.9}} \]

For Gravity driven and mixed flows (shaded purple in the flow regime map)

\[ \delta(x) = \frac{15.93 \times x^{0.26} \times (Ja_1/Pr_1)^{0.2684} \times (\rho_2/\rho_1)^{0.8065}}{Re_{in} \times (\mu_2/\mu_1)^{0.8426} \times (Fr_x^{-1})^{0.3891}} \]

\[ 0.75x_{FC} = \frac{2.69 \times Re_{in} \times (\rho_2/\rho_1)^{1.1695} \times (\mu_2/\mu_1)^{0.1085}}{(Ja_1/Pr_1)^{0.9911} \times (Fr_x^{-1})^{0.5334}} \]
Exit Condition Issue for Internal Condensing Flows
(Consider Partially Condensing Annular/Stratified Flows)

In the above thought experiment, one asks whether the exit condensate flow rate ($\dot{M}_{L-e}$) (or equivalent exit pressure) can be used to “control” the flow and achieve multiple quasi-steady solutions (not necessarily annular/stratified). In other words: are these flows “elliptic” (i.e. do these flows listen to both upstream and downstream conditions)?

- Yes!

Clearly, the above “control” is impossible for single-phase flows or adiabatic two-phase flows (with zero interfacial mass transfer) because, the information only travels downstream (i.e. they are parabolic flows).

Related Issues/Questions:
- What is the nature of the steady governing equations? Are they parabolic (as in single-phase or air-water flows)?
- Are there significant differences between gravity-driven and shear-driven flows?
• For an internal condensing flow, there is a unique steady “natural” annular/stratified (or “film” condensation) solution which can be realized in the absence of any “active” imposition of exit condition—i.e. when the set up allows the flow to seek its own exit condition.

• However one can “actively” impose different steady or quasi-steady exit conditions other than the “natural” one. This typically leads to other time dependent or quasi-steady solutions which may cause the flow regime to shift from annular stratified to non-annular (plug, slug, etc.) flows. This shows that the unsteady equations for these flows are “elliptic”—i.e. exit conditions matter. The impact is significant for shear driven flows and insignificant for gravity driven flows.

• For partial condensation, exit condition can be imposed either through control of the liquid exit mass flow rate or vapor exit mass flow rate—achieved by active pumping with the help of displacement pump P or P₁ shown in the figure above.

• The response of the flow to this controllability depends on:
  - Nature of the exit flow rate control function
  - Type of the annular/stratified flow - i.e. gravity driven or shear driven
Result 2 (contd.): Energy Dissipation in the Condenser

“On-Off” control with a mean that is near “natural”:
Results obtained by the computational tool

Annular flows realized through “on-off” control in this range correspond to the limited steady energy band associated with this control.

For “on-off” controls in the vicinity of steady “natural” energy consumption indicated above—nearby steady solutions exist. This makes PID control of exit flow rate possible – because the mean of the “on-off” control does not have to be exactly at the “natural.”
Continuous “Off-Natural” Control: Long time ($t \to \infty$) non-annular quasi-steady flows mean that dissipative energy results can only be conjectured. The conjectured result is:
Key Experimental Results for Partially Condensing Internal Condensing Flows

- Gravity driven flows were found to be “parabolic” and no exit conditions could be experimentally imposed.
- For shear driven flows, repeatable annular stratified “natural” cases were achieved for unspecified exit conditions.
- For shear driven flows, theoretical results for flow controllability through exit conditions are being currently experimentally investigated and the results are expected soon.
Key Experimental Results for Fully Condensing Internal Condensing Flows

- Gravity driven flows were found to be “parabolic” and no exit conditions could be experimentally imposed.
- For shear driven fully condensing flows, repeatable complex morphology flows were experimentally achieved.
- For shear driven flows, theoretical results for flow controllability through exit conditions are being currently experimentally investigated and the results are expected soon.