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Finite Element Solution for Ionized Fields in DC Electrostatic Precipitator

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Abstract: This paper presents the finite element solution for ionized field in the coaxial cylinders configuration during the positive corona discharge using COMSOL MULTIPHYSICS 3.3. The coupled differential equations describing monopolar DC corona are completely solved by using three application modes with the appropriate boundary conditions: PDE (General Form) mode for electric potential distribution, PDE (General Form) mode for space charge density equation, and PDE (Weak Form, boundary) mode used to impose the constraint for electric potential at coronating conductor boundary. The spacial distributions for the electric field, the electric potential, and the space charge density are computed and compared with the analytical solution.

The current-voltage characteristic, obtained by mean of the computational procedure, is in good agreement with the experimental data obtained on laboratory scale electrostatic precipitator.

Keywords: Ionized Field, Corona Discharge, Electrostatic Precipitator, COMSOL MULTIPHYSICS.

1. Introduction

The electrostatic precipitators (ESP) are the most widely used systems to reduce the particulate emissions from large industrial process plant. Essentially, electrostatic precipitators remove dust by charging the constituent particles by means of corona generated ions. Then the electrically charged particles move towards the collecting electrode under the effect of electric field present in the inter-electrode space, which is produced by the high voltage applied to the emitter electrode [1].

The determination of the electric field and current density distributions in inter-electrode space is further complicated by the presence of the corona phenomenon. The space charge distribution may be computed analytically only for particular symmetry arrangements under some simplifying assumptions [2]. Several models have been proposed for the calculation of the electric field and charge density distribution using the finite difference method [3], finite element techniques [4] which are usually combined with the method of characteristics (MOC) [5], boundary element method with MOC [6] and combined boundary element with finite difference method [7]. Also, alternative methodologies have been developed such as the R-functions and MOC [8], the donor cell method with finite elements [9], the charge simulation technique (CSM) [10], and a model that combines finite differences and finite elements [11]. In the present paper, the calculation of the electrical quantities is performed using a finite element software, COMSOL MULTIPHYSICS. The computed values are compared with those obtained analytically [12] to assess the accuracy of the obtained results. The calculated current-voltage characteristics are compared with measured ones carried out on a laboratory model of coaxial wire-cylinder electrostatic precipitator.

Therefore, the aim of the present work is to validate a numerical model and to show that modeling of electric field during the corona discharge can be successfully performed using COMSOL MULTIPHYSICS software.

2. Mathematical Model

The equations that constitute the mathematical description of the corona phenomenon are obtained from the following relations:

$$\vec{\nabla} \cdot \vec{E} = \rho/\varepsilon_0 \qquad (1)$$

(Gauss's law)

$$\vec{\nabla} \cdot \vec{J} = 0 \qquad (2)$$

(Current continuity condition)

$$\vec{J} = \rho \cdot \mu \cdot \vec{E} \qquad (3)$$

(Equation of current density)

$$\vec{E} = -\vec{\nabla}\varphi \qquad (4)$$

(Equation relating the electric field to the potential)

Where $\vec{J}$ is the current density vector, ρ the ionic charge density, ε_0 is permittivity of free space, μ the ionic mobility (assumed constant in this work), $\vec{E}$ the electric field, and φ the electric potential.

By substituting (4) into (1), we can obtain the familiar Poisson's equation:

$$\vec{\nabla} \cdot \vec{\nabla}\varphi = -\frac{\rho}{\varepsilon_0} \qquad (5)$$

and substituting (3) into (2), expanding the divergence, taking into account of (1) and (4) provides an equation that governs the evolution of charge density in inter-electrode space:

$$\vec{\nabla}\rho \cdot \vec{\nabla}\varphi - \frac{\rho^2}{\varepsilon_0} = 0 \qquad (6)$$

3. Boundary Conditions

From the previous equations (1-4) we obtained a single third-order nonlinear partial differential equation:

$$\vec{\nabla}.\left[\left(\vec{\nabla}.\vec{\nabla}\varphi\right)\vec{\nabla}\varphi\right] = 0$$

The solution requires three boundary conditions: (i) the potential on the coronating wire is equal to the applied voltage; (ii) the potential on the grounded electrode is zero; (iii) the electric field at the emitter electrode is derived from Kaptzov's assumption [13]: the corona discharge is assumed to be distributed uniformly over the surface of the wire. If the corona electrode has a potential above a certain value, called the onset level, the normal component of the electric field remains constant at the onset value E_0.

$E_0 = 31.02 \cdot 10^5 \cdot \alpha \cdot \delta\left(1 + \frac{0.0308}{\sqrt{r_w \cdot \delta}}\right)$, where the wire radius r_w is expressed in meters, α is the dimensionless surface roughness of electrode ($\alpha = 1$ for smooth surface) and δ is the relative density of air ($\delta = \frac{293}{273+T}\frac{P}{101325}$). Where T and P are the actual temperature and pressure of the air for which δ is to be calculated.

The equations (5 & 6) are solved for the wire-cylinder ESP with resort to some simplifying assumptions:

- The ion charge in the inter-electrode space is monopolar and in steady-state conditions.
- The thickness of the ionization zone around the emitter is negligible.
- The mobility of ions is constant (independent of electric field intensity).

4. Results

The coaxial wire-cylinder electrostatic precipitator that has been studied consisted of a wire of radius $r_w = 0.15\,mm$ and a cylinder of radius $r_c = 17\,mm$ (Figure 1).
Due to cylindrical symmetry of the problem, the distributions of the potential φ, electric field $\vec{E}$ and space charge density ρ are functions of only the radial coordinate r. The problem is reducible to that in one dimension.

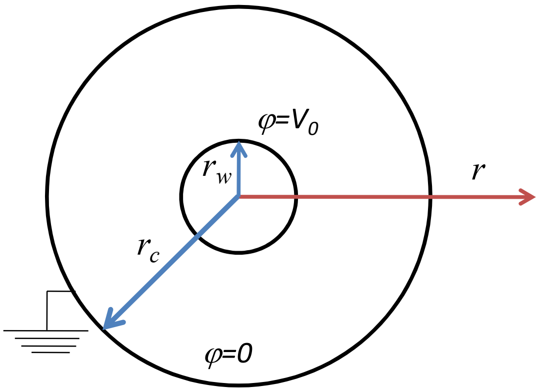

Figure 1. Cross section of coaxial wire-cylinder configuration

The numerical simulations were carried out using COMSOL MULTIPHYSICS 3.3 version. Equation for electric potential (5) was solved using PDE (General Form) mode with one Dirichlet boundary condition:

$$\varphi = 0 \;\; at \;\; r = r_c$$

and one Neumann boundary condition:

$$E = E_0 \;\; at \;\; r = r_w$$

Equation for space charge density (6) was solved using PDE (General Form) mode with one Dirichlet boundary condition at inner conductor:

$$\rho = rhoc$$

and point equation for $rhoc$ using PDE (Weak Form, boundary application) mode with desactivated on outer electrode boundary. $rhoc$ becomes a Lagrange Multiplier lm that is used to impose the constraint $\varphi = V_0$ at the surface of the wire, which is implemented using the weak formulation: bnd.weak= $\{rhoc\_test * (V0 - \varphi)\}$.